$$\cos(\psi - \alpha_2) = \eta(\alpha_2), \tag{2.27}$$

and $\psi = \psi(\varphi_2)$ in (2.26) and (2.27). Multiplying the first equation of (2.26) by $\cos\alpha_2$ and the second one by $\sin\alpha_2$ and adding them, we obtain with (2.27) taken into account a transcendental equation for the determination of α_2 for different values of z_2 and α:

$$\eta(\alpha_2) + t_2 \sin(\alpha + \alpha_2) + z_2 \cos 2\alpha \sin(\alpha - \alpha_2) = 0. \tag{2.28}$$

We note that when calculating α_1 from (2.10) and α_2 from (2.28) one should take only those values which belong to the sector $\Sigma_5\Sigma_6$. Having determined the position of the wave α_2 from the formula (2.28), we find its propagation velocity c_2 from a relationship similar to (2.3). The equality of (2.24) with $\varphi = \varphi_2$ to the first expression of (2.13) serves as the criterion for the correctness of the numerical calculations for α_2. The problem has been solved.

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ELASTIC STRESSES NEAR JOINTS OF BOUNDARIES OF CRYSTALLITES SUBJECTED TO SELF-DISTORTIONS

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UDC 539.4

1. The strength and plasticity of solids depends to a large extent on their superatomic structure. For polycrystalline materials, these important structural elements include crystallites (grains), crystallite boundaries, and joints of crystallite boundaries (JCB). Recently, a number of investigators established that JCB (or joints of boundaries of fragments) can be locations for generation of microcracks both with active deformation [1, 2] and in the creep regime [3, 4]. The concentration of thermoelastic stresses near JCB often causes information of microscopic cracks in ceramic materials [5]. Elastic stresses, arising near JCB, play an important role in recrystallization processes [4] and superplastic deformation [6].

The concentration of elastic stresses near JCB could be a result of several factors: elastic inhomogeneity (or anisotropy) of the material, high-temperature slipping along crystallite boundaries and, finally, self-distortion of crystallites. Stresses near sharp elastic inhomogeneities were examined in [7]. The results in [8] permit estimating the elastic stresses related to slipping along intersecting crystallite boundaries. In this work, we examine the problem of finding the distribution of elastic stresses near JCB in the third case, when the joining crystallites undergo self-distortions. In this case, self-distortions are taken to mean any (plastic, thermal, magnetostrictive, etc.) distortions of crystallites of a nonelastic nature. It is convenient to calculate the stresses by methods of the continuum theory of dislocations and disclinations [9-11]. Internal elastic stresses can be represented as a superposition of fields of elastic stresses of distributed dislocations.

2. Let us examine n wedge-shaped crystallites with planar boundaries $OP^{(m)}$ ($m = 1, 2 \ldots n$), joining along the z axis of a Cartesian coordinate system x, y, z (Fig. 1). The z axis is perpendicular to the plane of the figure. We shall assume that the crystallites are infinite along the z axis and are subjected to homogeneous self distortion $\beta_{ik}^{(m)}$, where the index m corresponds to the number of the crystallite. In the general case, the distortions

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$\beta_{ik}^{(m)}$ are incompatible and the system of crystallites as a whole will be found in a stressed state, independent of z. The self-distortion of the system of crystallites β_{ik} can be represented as a sum

$$\beta_{ik} = \sum_m \beta_{ik}^{(m)} \delta(V^{(m)}), \tag{2.1}$$

where [9]

$$\delta(V^{(m)}) = \int_{V^{(m)}} \delta(\mathbf{r} - \mathbf{r}')\, dV'.$$

Here $\delta(\mathbf{r})$ is a delta function; r, radius-vector; $V^{(m)}$, region occupied by the m-th crystallite. The state of the body with given self-distortion corresponds to a dislocation state with a tensor dislocation density α_{pl}, defined by the relation [9-11]:

$$\alpha_{pl} = \varepsilon_{prk} \beta_{kl,r}, \tag{2.2}$$

where ε_{prk} is the unit antisymmetric tensor, and the index following the comma indicates differentiation with respect to the Cartesian coordinate (indices 1, 2, 3 correspond to x, y, z). Substituting (2.1) into (2.2), we obtain

$$\alpha_{pl} = \sum_m \alpha_{pl}^{(m)},$$

where $\alpha_{pl}^{(m)}$ is the dislocation density tensor, corresponding in (2.1) to the term $\beta_{ik}^{(m)}\delta(V^{(m)})$: $\alpha_{pl}^{(m)} = \varepsilon_{prk}\beta_{kl}^{(m)}\delta_{,r}(V^{(m)})$. Using the rule for differentiating the three-dimensional delta function [9] and the equation for the boundary $OP^{(m)}$ in the form $k^{(m)}$ is the slope of the straight line) $y = k^{(m)}x$, we find $\alpha_{pl}^{(m)} = -\varepsilon_{prk}\beta_{kl}^{(m)}\delta_r(S^{(m)})$, where $\delta_k(S^{(m)}) = \int_{S^{(m)}} \delta(\mathbf{r} - \mathbf{r}')\, dS_k'$ is a two-dimensional delta function [9]; $S^{(m)}$ is the surface of the boundary $OP^{(m)}$. As is evident from (2.2), the dislocation density turns out to be concentrated on the surface of the boundaries $S^{(m)}$. In addition, we shall assume that the m-th crystallite has boundaries $S^{(m)}$, $S^{(m-1)}$, with $m \neq 1$ and boundaries $S^{(n)}$, $S^{(1)}$ for m = 1. The surface $S^{(m)}$ serves as a boundary between two contiguous regions $V^{(m-1)}$, $V^{(m)}$ and, in addition $\delta_{,r}(V^{(m-1)}) = -\delta_r(S^{(m)})$, $\delta_{,r}(V^{(m)}) = \delta_r(S^{(m)})$.

Taking this into account in the summation of $\alpha_{pl}^{(m)}$, we obtain for α_{pl} $\left(\delta(S^{(m)}) = \int_{S^{(m)}} \delta(\mathbf{r} - \mathbf{r}')\, dS'\right)$

$$\alpha_{pl} = \sum_m A_{pl}^{(m)} \delta(S^{(m)}), \tag{2.3}$$

where $A_{pl}^{(m)} = \varepsilon_{prk}\Delta\beta_{kl}^{(m)} n_r(S^{(m)})$; $n_r(S^{(m)})$ is the normal to $S^{(m)}$; $\Delta\beta_{kl}^{(m)} = \beta_{kl}^{(m)} - \beta_{kl}^{(m-1)}$ is the jump of the self distortions in crossing $S^{(m)}$ from the region $V^{(m-1)}$ into the region $V^{(m)}$.

3. We shall proceed to calculate the distribution of internal elastic stresses near JCB. For simplicity, we shall assume that the crystallites are elastically isotropic and have identical elastic constants. It is convenient to calculate the stresses separately for each planar distribution of dislocations in (2.3) $A_{pl}^{(m)}\delta(S^{(m)})$ in the intrinsic coordinate system $x^{(m)}$, $y^{(m)}$, $z^{(m)}$. The coordinates x, y, z and $x^{(m)}$, $y^{(m)}$, $z^{(m)}$ have a common origin, the $z^{(m)}$ axis is oriented along the z axis, the $x^{(m)}$ axis lies in the plane $OP^{(m)}$, while the $y^{(m)}$ axis is normal to $OP^{(m)}$, as shown in Fig. 2. In what follows, all quantities in the intrinsic system of coordinates will be denoted by a bar above the quantity. In the intrinsic coordinate system, the vector normal to $S^{(m)}$ has components (0, 1, 0) so that the tensor $\bar{A}_{pl}^{(m)} = \varepsilon_{prk}\Delta\bar{\beta}_{kl}^{(m)}\bar{n}_r \cdot (S^{(m)})$ has the following nonzero components:

$$\bar{A}_{11}^{(m)} = \Delta\bar{\beta}_{31}^{(m)},\ \bar{A}_{12}^{(m)} = \Delta\bar{\beta}_{32}^{(m)},\ \bar{A}_{13}^{(m)} = \Delta\bar{\beta}_{33}^{(m)},$$
$$\bar{A}_{33}^{(m)} = -\Delta\bar{\beta}_{13}^{(m)},\ \bar{A}_{32}^{(m)} = -\Delta\bar{\beta}_{12}^{(m)},\ \bar{A}_{31}^{(m)} = -\Delta\bar{\beta}_{11}^{(m)}.$$

In what follows, we shall calculate the elastic stresses from each component $\bar{A}_{ij}^{(m)}$, denoting the components of the elastic stress tensor by additional upper indices, for example, $\bar{\sigma}_{xx}^{ij(m)}$. For simplicity, we shall drop the index m in the coordinates. The fields corresponding to $\bar{A}_{33}^{(m)}\delta(S^{(m)})$ are equivalent to elastic fields created by screw dislocations

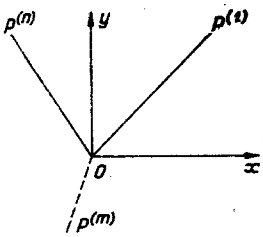

Fig. 1

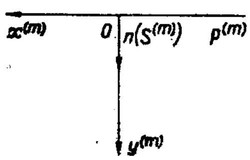

Fig. 2

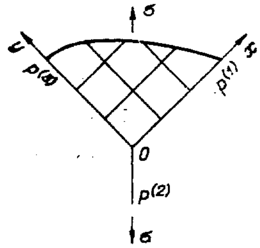

Fig. 3

parallel to the z axis with Burgers vector b = (0, 0, 1) and distributed with density $-\Delta\bar{\beta}_{13}^{(m)}$ along the surface $S^{(m)}$. Integrating the known stresses of separate screw dislocations [10], we obtain for the nonzero stress components

$$\bar{\sigma}_{xz}^{33(m)} = \frac{\mu}{2\pi}\Delta\bar{\beta}_{13}^{(m)}\left(\frac{\pi}{2} - \arctan\frac{x}{y}\right),\ \bar{\sigma}_{yz}^{33(m)} = \frac{\mu}{2\pi}\Delta\bar{\beta}_{13}^{(m)}\ln\left(\frac{\rho}{R_0}\right),$$

where μ is the shear modulus, $\rho = \sqrt{x^2 + y^2}$; and R_0 is the cutoff radius. Some of the integrals, calculated along the complete surface $S^{(m)}$, diverge. For this reason, it is necessary to introduce the cutoff radius R_0, which in the problem being examined, can be set equal to the average linear size of crystallites in a specific polycrystalline material. In the expressions presented, only the leading terms taking into account $\rho \ll R_0$ are retained.

The elastic fields, related to $\bar{A}_{31}^{(m)}\delta(S^{(m)})$, are equivalent to fields created by edge dislocations parallel to the z axis with Burgers vector b = (1, 0, 0), distributed with constant density $-\Delta\bar{\beta}_{11}^{(m)}$ along the surface $S^{(m)}$. Integrating the known stresses from separate edge dislocations [10], we obtain for the nonzero stress components

$$\bar{\sigma}_{xx}^{31(m)} = \frac{\mu}{2\pi(1-\nu)}\Delta\bar{\beta}_{11}^{(m)}\left(\frac{\pi}{2} - 2\arctan\frac{x}{y} + \frac{xy}{\rho^2}\right),\ \bar{\sigma}_{yy}^{31(m)} = -\frac{\mu}{2\pi(1-\nu)}\Delta\bar{\beta}_{11}^{(m)}\left(\frac{xy}{\rho^2}\right),$$

$$\bar{\sigma}_{zz}^{31(m)} = \frac{\mu\nu}{\pi(1-\nu)}\Delta\bar{\beta}_{11}^{(m)}\left(\frac{\pi}{2} - \arctan\frac{x}{y}\right),$$

$$\bar{\sigma}_{xy}^{31(m)} = \frac{\mu}{2\pi(1-\nu)}\Delta\bar{\beta}_{11}^{(m)}\left(\frac{y^2}{\rho^2} + \ln\frac{\rho}{R_0}\right),$$

where ν is Poisson's coefficient.

The component $\bar{A}_{32}^{(m)}\delta(S^{(m)})$ of the dislocation density tensor gives rise to elastic fields that are equivalent to fields created by edge dislocations parallel to the z axis with Burgers vector b = (0, 1, 0), distributed with constant density $-\Delta\bar{\beta}_{12}^{(m)}$ along the surface $S^{(m)}$. Integrating the known stresses from isolated dislocations of the type indicated [10] yields the following nonzero elastic stress components

$$\bar{\sigma}_{xx}^{32(m)} = \frac{\mu}{2\pi(1-\nu)}\Delta\bar{\beta}_{12}^{(m)}\left(\frac{y^2}{\rho^2} + \ln\frac{\rho}{R_0}\right),\ \bar{\sigma}_{yy}^{32(m)} = \frac{\mu}{2\pi(1-\nu)}\Delta\bar{\beta}_{12}^{(m)}\left(-\frac{y^2}{\rho^2} + \ln\frac{\rho}{R_0}\right),$$

$$\bar{\sigma}_{zz}^{32(m)} = \frac{\mu\nu}{\pi(1-\nu)}\Delta\bar{\beta}_{12}^{(m)}\left(\ln\frac{\rho}{R_0}\right),\ \bar{\sigma}_{xy}^{32(m)} = -\frac{\mu}{2\pi(1-\nu)}\Delta\bar{\beta}_{12}^{(m)}\left(\frac{xy}{\rho^2}\right).$$

The elastic field from the component $\bar{A}_{11}^{(m)}\delta(S^{(m)})$ of the dislocation density tensor, which is equivelant to fields from semiinfinite screw dislocations parallel to the x axis with Burger's vector b = (1, 0, 0), distributed with constant density $\Delta\bar{\beta}_{31}^{(m)}$ along the surface $S^{(m)}$, has a very simple form. Integrating the stresses from the semiinfinite screw dislocation [10], we obtain for the only nonzero component of the elastic stress tensor

$$\bar{\sigma}_{xz}^{11(m)} = \frac{\mu}{2\pi}\Delta\bar{\beta}_{31}^{(m)}\left(\frac{\pi}{2} - \arctan\frac{x}{y}\right).$$

The contribution from the component $\bar{A}_{12}^{(m)}\delta(S^{(m)})$ of the dislocation density tensor is determined by integrating elastic stresses from semiinfinite edge dislocations parallel to the x axis with Burgers vector b = (0, 1, 0) distributed with constant density $\Delta\bar{\beta}_{32}^{(m)}$ along the surface $S^{(m)}$. The nonzero components of the stress tensor have the form

$$\bar{\sigma}_{xx}^{12(m)} = -\frac{\mu}{2\pi(1-\nu)}\Delta\bar{\beta}_{32}^{(m)}\left(\frac{xy}{\rho^2}\right),\ \sigma_{xz}^{12(m)} = = \frac{\mu}{2\pi(1-\nu)}\Delta\bar{\beta}_{32}^{(m)}\left[1+(1-\nu)\ln\frac{\rho}{2R_0}\right],$$

$$\bar{\sigma}_{zy}^{12(m)} = \frac{\mu}{2\pi(1-\nu)}\Delta\bar{\beta}_{32}^{(m)}\left[\frac{\rho^2}{x^2}\left(\frac{\pi}{2}-2\arctan\frac{x}{y}\right)+2\frac{y\rho^2}{x^3}\right].$$

Finally, the contribution from the components $\bar{A}_{13}^{(m)}\delta(S^{(m)})$ of the dislocation density tensor is determined by integrating the elastic stresses from semiinfinite edge dislocations parallel to the x axis with Burgers vector b = (0, 0, 1) distributed with constant density $\Delta\bar{\beta}_{33}{}^{(m)}$ over the surface $S^{(m)}$. For the nonzero components of the stress tensor, we obtain

$$\bar{\sigma}_{xx}^{13(m)} = \frac{\mu}{2\pi(1-\nu)}\Delta\bar{\beta}_{33}^{(m)}\left(\frac{xy}{\rho^2}-2\nu\arctan\frac{x}{y}-3\pi\nu\right),$$

$$\bar{\sigma}_{yy}^{13(m)} = \frac{\mu}{2\pi(1-\nu)}\Delta\bar{\beta}_{33}^{(m)}\left(\frac{x^4-y^4-x^2y^2+xy^3-yx^3}{x^2\rho^2}+\pi\frac{y^3}{x^2\rho}\right),$$

$$\bar{\sigma}_{zz}^{13(m)} = \frac{\mu}{2\pi(1-\nu)}\Delta\bar{\beta}_{33}^{(m)}\left(\frac{y^2-xy}{x^2}\arctan\frac{x}{y}-2\frac{y^3}{x^3}+\frac{\pi}{2}\,\frac{x^2-y^2}{x^2}\right),$$

$$\bar{\sigma}_{xy}^{13(m)} = \frac{\mu}{2\pi(1-\nu)}\Delta\bar{\beta}_{33}^{(m)}\left(\nu\ln\frac{\rho}{2R_0}+\frac{y^2}{\rho^2}\right).$$

The complete expressions for the stresses σ_{ij} created by all components of the dislocation density tensor $\bar{A}_{kl}^{(m)}\delta(S^{(m)})$, taking into account the contribution of all surface distributions, can be written in the form of a sum

$$\sigma_{ij} = \sum_{k,l,m}\sigma_{ij}^{kl(m)},$$

where $\sigma_{ij}^{kl(m)} = \sum_{s,t} a_{is}^{(m)}a_{jt}^{(m)}\bar{\sigma}_{st}^{kl(m)}$ are the components of the stress tensor in the laboratory system of coordinates; $a_{is}{}^{(m)}$ is the coordinate transformation matrix in going from the intrinsic system of coordinates to the laboratory system. It should be noted that the stresses $\bar{\sigma}_{ij}^{kl(m)}$ separately may not satisfy the equilibrium equations $\sum_j \bar{\sigma}_{ij,j}^{kl(m)} = 0$, since only the total stresses have physical significance [12].

As an application of the results obtained above, we shall analyze the internal stresses that can arise near triple JCB during plastic deformation of a polycrystal. We shall examine three crystallites joining along the z axis, as shown in Fig. 3 (here, we use the same notation as in Fig. 1). Let the first upper crystallite undergo plastic flow by slipping along the surfaces parallel to the x and y axes (in Fig. 3, the surfaces are shown by straight lines), under the action of external stretching stresses σ, while the remaining crystallites remain in the elastic state (for them, the Shmidt factor is not favorable [4]). If we assume that the plastic deformation is homogeneous, then the self- (plastic) distortion of the first crystallite has the following nonzero components: $\beta_{xy}^{(1)} = \beta_{yx}^{(1)} = \gamma$, where γ is the magnitude of the deformation stretching. In this case, from the preceding equations, it follows that near a triple JCB there arise elastic stresses that are equivalent to stresses from a biaxial dipole (with arm R_0) of wedge-shaped disclinations [11]. Identical normal stretching stresses will act in any plane passing throuth the z axis

$$\sigma_n(\rho) = [\mu/2\pi(1-\nu)]2\gamma\ln(R_0/\rho).$$

The generation of microcracks in such a stress field was examined previously in [13] and the condition for generation of microcracks can be written in the form

$$\gamma = \gamma_+ = 2[2\pi(1-\nu)\Gamma/\mu a]^{1/2}[\ln(4R_0/a)]^{-1},$$

where Γ is the surface rupture energy; a is the lattice parameter; and, γ_+ is the critical degree of deformation.

If we set for estimates $[4\pi(1-\nu)/\mu a]\Gamma \sim 1$ [13], then microcracks will be generated after the critical deformation $\gamma_+ \sim \sqrt{2}[\ln(4R_0/a)]^{-1}$, which decreases with increasing size of crystallites R_0. The behavior of polycrystallites (flow occurs only in some favorably oriented crystallites) examined above can be observed for $\sigma < \sigma_+$, i.e., when external stresses do not exceed the macroscopic yield stress σ_+. For σ_+, the Petch–Hall dependence [4] on the size of crystallites R_0 of the form $\sigma_+ = \sigma_0 + kR_0^{-1/2}$ is valid, where σ_0, and k are some material constants. Comparing the dependence for $\gamma \sim \sigma/\mu$ and γ_+, σ_+, we can see that the conditions $\gamma = \gamma_+$ and $\sigma < \sigma_+$ can be satisfied simultaneously for small crystallite sizes R_0. This result agrees well with experimental observations [2], in which cracking (stratification) begins after a structure with very small crystallite sizes R_0 begins to fragment during plastic deformation.

The aforementioned concerns materials subjected to active deformation and capable of hardening. We shall now examine the case when the polycrystalline material deforms without significant hardening (the high-temperature creep regime or superplastic deformation [4, 14]). Taking into account the fact that in this case all crystallites deform plastically, it is necessary to keep in mind the relative (difference) deformation of the crystallite $\Delta\gamma$, which is defined as the difference between the plastic deformation of a crystallite and the plastic deformation of the surroundings. Assuming a power-law dependence of the rate of deformation on stress [4, 14], we shall write for the rate of change $d\Delta\gamma/dt$ an equation of the form

$$\frac{d}{dt}\Delta\gamma = \gamma_0\left(\frac{\sigma - \varkappa\Delta\gamma}{m_1}\right)^p - \gamma_0\left(\frac{\sigma}{m_2}\right)^p,$$

where p is the exponent; γ_0 and $\varkappa$ are constants; and m_1 and m_2 are effective orientational factors for the crystallite being examined and its surroundings. The first term on the right has the meaning of a rate of plastic deformation of the crystallite taking into account the constriction of the environment (i.e., reverse stress $\varkappa\Delta\gamma$), and the second term is the rate of plastic deformation of the environment. The difference plastic deformation of the crystallite $\Delta\gamma$ increases, if $m_1 < m_2$, i.e., if the crystallite has more favorable orientation relative to the stretching axis than the environment. However, $\Delta\gamma$ stabilizes by the constrictive action of the surroundings (stresses $\varkappa\Delta\gamma$). If for estimates, we assume that the crystallite is spherically shaped, then, following Eshelby's method [15], we obtain for $\varkappa$

$$\varkappa = \frac{2}{9}\,\frac{(1-2\nu)}{(1-\nu)}\left(2\mu + \frac{E\nu}{(1+\nu)(1-2\nu)}\right) + \frac{4}{45}(7-5\nu)\,\mu,$$

where E is Young's modulus. For $t \to \infty$ $\Delta\gamma \to \Delta\gamma_{max} = (1/\varkappa)(1 - m_1/m_2)\sigma$. Let $\Delta\gamma_{max} \gg \gamma_+$, the critical deformation for generating microcracks (or pores). Then, right up to the point at which $\Delta\gamma$ attains the critical value γ_+, the reverse stresses $-\varkappa\Delta\gamma$ can be neglected, and λ, the ratio of the rate of difference deformation $d\Delta\gamma/dt$ to the rate of deformation of the surroundings $(\sigma/m_2)^p$ to the rate of deformation of the surroundings $(\sigma/m_2)^p$ is expressed by the equation $\lambda = (m_2/m_1 - 1)^p$. For σ = const, the magnitude of the overall plastic deformation of the specimen (we assume that it equals the plastic deformation of the surroundings) up to the time of failure $\gamma_- = \gamma_+/\lambda$ and for γ_+ = const increases with decreasing λ. Large values of γ_- are possible for large p, if $m_2/m_1 > 2$, and for small p if $1 \leqslant m_2/m_1 < 2$. The quantity 1/p is called the coefficient of rate sensitivity [14] and, in addition, for superplastic deformation, large values of both γ_- and 1/p are characteristic. This agrees with the qualitative analysis presented above, if we assume that $1 \leqslant m_2/m_1 < 2$. Such an assumption, apparently, is reasonable, since superplastic deformation is observed at high temperatures, when multiple slipping increases and when the spread in the effective orientational factors must be small (ratio m_2/m_1 differs little from unity). This is confirmed, in particular, by the absence of texture accompanying superplastic deformation [14].

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PROBLEM OF ESTIMATING THE CREEP STRENGTH UNDER STEP LOADING

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In many investigations of the work of structures under variable stress under prolonged high-temperature action conditions, the main question is the possibility of estimating the rupture time from test results under constant stresses. The rule of linear summation of the partial times, proposed in [1] to analyze test results under variable temperature, is ordinarily utilized as the simplest and best known hypothesis. We consider the case when the stress σ_1 in the specimen and the effective temperature t_1 changes by a jump to σ_2 and remains constant for a time t_2 until rupture at the time $t^* = t_1 + t_2$. We write the sum of the partial times in the form

$$A = t_1/t_1^* + t_2/t_2^*. \tag{1}$$

In case the principle of linear summation is satisfied

$$A \equiv 1. \tag{2}$$

Here t_1^* (or t_2^*) is understood to be the time to fracture for stresses σ_1 (or σ_2) invariant during the testing. Many investigations confirm the rule (2) to some extent, however, systematic deviations are observed in a significant quantity of papers, which are outside the boundaries of the natural spread. For certain materials a deviation of A from 1 to one side is hence characteristic, independently of the test parameters, while for other materials the quantity A is greater or less than 1 depending on the sign of the difference $(\sigma_1 - \sigma_2)$.

The behavior of steel EI388 at 600°C was investigated in [2] for $\sigma_1 > \sigma_2$ and $\sigma_1 < \sigma_2$ for small changes in the stress $(|\sigma - \sigma_2|/\sigma_1 < 0.06)$, and the tests exhibited a significant one-sided deviation from the law (2): the mean value of A was $A_0 = 0.72$. A model permitting the description of the deviation of A from 1 to one side, independently of the sign of the difference $(\sigma_1 - \sigma_2)$, is proposed below.

The concept of a mechanical equation of state, proposed in [3], is used with a system of kinetic equations within the framework of the mechanics of continuous media to describe the creep of metals, to determine the parameters characterizing the state under consideration. One structural parameter $\omega(t)$ which is a certain measure of the "spalling" of the material, is utilized most frequently to describe the creep strength. A value of ω from the range $0 \leqslant \omega \leqslant 1$, is ascribed to each "spalling" state, where $\omega = 0$ corresponds provisionally to the undamaged material, and $\omega = 1$ corresponds to the time of rupture t^*.

It is known that the nature of rupture for a number of materials at the identical temperature can be qualitatively distinct depending on the stress level. At high stresses the

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